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CALIBRATION OF FLOW NOZZLES USING TRAVERSING PITOT-STATIC PROBES

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ABSTRACT

A description is given of a method for calibrating flow nozzles used in compressor testing which is insensitive to flow variations during a survey. A computer program to reduce hand-recorded data from manometers is described which uses a compressible flow expression whose derivation is included in the first appendix. This appendix gives forms of the statement of continuity along a streamline when "limiting" values of the density and velocity are used to non-dimensionalize these parameters. This technique has been found to simplify the writing of computer programs for analyzing measurements from turbomachines, and to generally simplify analyses of flows that can be taken to be adiabatic.

The method used to integrate experimental data taken over arbitrary intervals, which is of general use, is described and a listing of the subroutine is given in a second appendix.

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Calibration of Flow Nozzles Using Traversing Pitot-Static Probes

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I. INTRODUCTION

In establishing the performance of turbomachines, and in propulsion system testing generally, the flow rate through the machine must be measured. If a "flow nozzle" is used in a standard installation, tables and charts are available, such as in Reference 1 and Reference 2, which are stated to give an accuracy of one or two percent in relating measured pressures to the mass flow rate. If better accuracy is desired, or if the installation is not within the specified standards, then the relationship between the pressure measurements and the flow rate must be established by calibration. Since there is no primary standard of measurement for large flow rates of gases, this is usually done by integrating measured distributions of mass flux inferred from pitot-static probe measurements. In order to obtain sufficient accuracy, water column U-tubes must be tediously hand recorded as the probe is traversed and the data carefully reduced and integrated. The purpose of this report is to document a self-consistent method of calibrating a flow nozzle which includes effects of compressibility explicitly, which minimizes inaccuracies arising from flow changes during a probe traverse, and which employs computer reduction of the hand-recorded manometer data. The calibration method establishes the "coefficient of discharge" for the nozzle (including the "velocity of approach factor" of Reference 1), which is expected to be a function of the Reynolds number (or flow rate). A "thermal expansion coefficient," which is omitted here, should be introduced for operation at significantly different temperatures.

II. ANALYSIS

The geometry and instrumentation are shown in Fig. 1. As the probe is traversed across the nozzle, at any station the mass flux in the annulus of depth dR is given by, using Eq. A(21) derived in Appendix A,

$$d\dot{m} = \sqrt{\frac{2p_p(p_p - p_1)}{R_g T_t}} \left[1 - \frac{3}{4\gamma}\left(\frac{p_p - p_1}{p_p}\right)\right] 2\pi R . dR \tag{1}$$

where we have neglected terms of second and higher order in $\left(\frac{p_p - p_1}{p_p}\right)$.

Here:

p_p = probe pitot pressure

p_1 = probe static pressure

R = radial position

R_g = gas constant

T_t = flow stagnation temperature

$\dot{m}$ = mass flow rate

The units are omitted at this stage.

If the nozzle shown in Fig. 1 behaved ideally, in a one-dimensional manner, then the flow rate would be given by

$$\dot{m}_i = \sqrt{\frac{2p_{f1}\ (p_{f1} - p_s)}{R_g T_t}} \left[1 - \frac{3}{4\gamma}\ \left(\frac{p_{f1} - p_s}{p_{f1}}\right)\right]\ \pi R_0^2 \tag{2}$$

where:

$\dot{m}_i$ = total mass flow rate for a one-dimensional flow

p_{f1} = upstream flange (stagnation) pressure

p_s = throat (static) pressure

R_0 = nozzle throat radius

The "coefficient of discharge," C, of the nozzle is defined as the ratio of the actual flow rate to the ideal flow rate for the same pressure measurements. The flow rate is then given by

$$\dot{m} = \int_{\text{nozzle}} d\dot{m} = C \cdot \dot{m}_i \tag{3}$$

where $\dot{m}_i$ is obtained from Eq. (2).

From Eq. (3) and Eq. (1),

$$C = \frac{1}{\dot{m}_i} \int_0^{R_0} \sqrt{\frac{2p_p(p_p - p_1)}{R_g T_t}} \left[1 - \frac{3}{4\gamma}\left(\frac{p_p - p_1}{p_1}\right)\right] 2\pi R . dR. \tag{4}$$

Since $\dot{m}_i$ is a constant (ideally) during a calibration traverse, it can be taken inside the integral so that, using Eq. (2),

$$C = 2.\int_0^1 \sqrt{\frac{p_p(p_p - p_1)}{p_{f1}(p_{f1} - p_s)}} \left[\frac{1 - \frac{3}{4\gamma}\left(\frac{p_p - p_1}{p_p}\right)}{1 - \frac{3}{4\gamma}\left(\frac{p_{f1} - p_s}{p_{f1}}\right)}\right] . \left(\frac{R}{R_0}\right) .d\left(\frac{R}{R_0}\right) \tag{5}$$

or, writing $\bar{R} = R/R_0$,

$$C = 2\int_0^1 F.\bar{R}.d\bar{R} \tag{6}$$

where

$$F = \sqrt{\frac{p_p(p_p - p_1)}{p_{f1}(p_{f1} - p_s)}} \left[\frac{1 - \frac{3}{4\gamma}\left(\frac{p_p - p_1}{p_p}\right)}{1 - \frac{3}{4\gamma}\left(\frac{p_{f1} - p_s}{p_{f1}}\right)}\right] \tag{7}$$

The integrand F can be seen to be the ratio of the local mass flux to the ideal local mass flux. While there may be variations in the total flow rate during the traverse of the probe, the value of F at a particular location is not likely to be affected. The method should then be to measure

F at values of $\bar{R}$, and to integrate the product $2(F \times \bar{R})$ over the nozzle radius. Note that the product $(F \times R) \rightarrow 0$ at the center of the nozzle and also at the wall, so that a linear extrapolation from the last data point taken near to the wall and near to the center can be made.

III. METHOD

Small differences in pressure, denoted by Δ , are read on water U-tube manometers.

We define,

$$\begin{aligned} \Delta_1 &= (p_p - p_1)/.07355 \\ \Delta_s &= (p_{f1} - p_s)/.07355 \\ \Delta_p &= (p_{f1} - p_p)/.07355 \\ \Delta_{sf} &= (p_{f1} - p_{f2})/.07355 \\ \Delta_f &= (p_a - p_{f1})/.07355 \end{aligned} \tag{8}$$

where p_a is the atmospheric pressure, and all Δ's are in units of inches of water, and all p's are in inches of mercury.

Figure 1 shows the manometer connections. The differences in Eq. (8) are then obtained (for this arrangement) from

$$\begin{aligned} \Delta_1 &= H_{10} - H_9 \\ \Delta_s &= H_3 - H_4 \\ \Delta_p &= H_5 - H_6 \\ \Delta_{sf} &= H_7 - H_8 \\ \Delta_f &= H_2 - H_1 \end{aligned} \tag{9}$$

where H_n is the height of the liquid level in column n in inches. From Eq. (8), for a room temperature of $70^{\circ}F$,

$$p_{f1} = p_a - 0.07355\ \Delta_f \tag{10}$$

and

$$p_p = p_c - 0.07355\ \Delta_p \tag{11}$$

No correction is included here for changes in the manometer fluid temperature.

The probe position is given in terms of a linear scale reading, S , by

$$\bar{R} = \frac{S-S_0}{R_0} \tag{12}$$

where S_0 is the scale reading when the probe is at the center. S_0 is determined either by physical inspection, or by assuming that the profiles near the wall of the nozzle are symmetrical with respect to the nozzle centerline. R_0 must have the units of S .

The function of pressure given by Eq. 7 is then

$$F = \frac{\left[1 - \frac{3}{4\gamma} \cdot \frac{\Delta_1}{p_p} \cdot 0.07355\right]}{\left[1 - \frac{3}{4\gamma} \cdot \frac{\Delta_s}{p_{f1}} \cdot 0.07355\right]} \sqrt{\frac{p_p}{p_{f1}} \frac{\Delta_1}{\Delta_s}} \tag{13}$$

The probe is traversed across the nozzle, and the water columns and the position scale reading are recorded at chosen intervals. At least five points are taken in each wall boundary layer, and the spacing is increased in the core of the flow to give a total of at least 40 points.

IV. DATA REDUCTION

The data is reduced using the IBM 360 computer. The program designated "BOX34NØZ" accepts the primary data on cards and outputs the coefficient of discharge and plots of the distribution of the function F .

The data cards are as follows

	Quantity	Program Notation	Format
CARD 1	Date	NDAY, NMTH, NYR	3I2
	Run No.	N SCAN	I4
	RPM:	NRPM	I10
	Throttle Setting	ØRFICE	I10
	Atmos. Pressure:	PATMØS	F10.2
	Atmos. Temperature	TATMØS	F10.1
	Scale Reading on Centerline	SZERØ	F10.3
	Nozzle Radius	RZERØ	F10.4
CARD 2	No. of Points:	N	I2
CARD 3 : CARD (2+N)	Scale Reading:	S(I)	F10.3
	Manometer Reading:	H1(I),H2(I),..H10(I)	10F6.2

The program calculates Δ's from Eq. 9, p_{fl} and p_p from Eq. 10 and Eq. 11, and the function F from Eq. 13. The normalized radius is calculated from Eq. 12, and the product is formed with F .

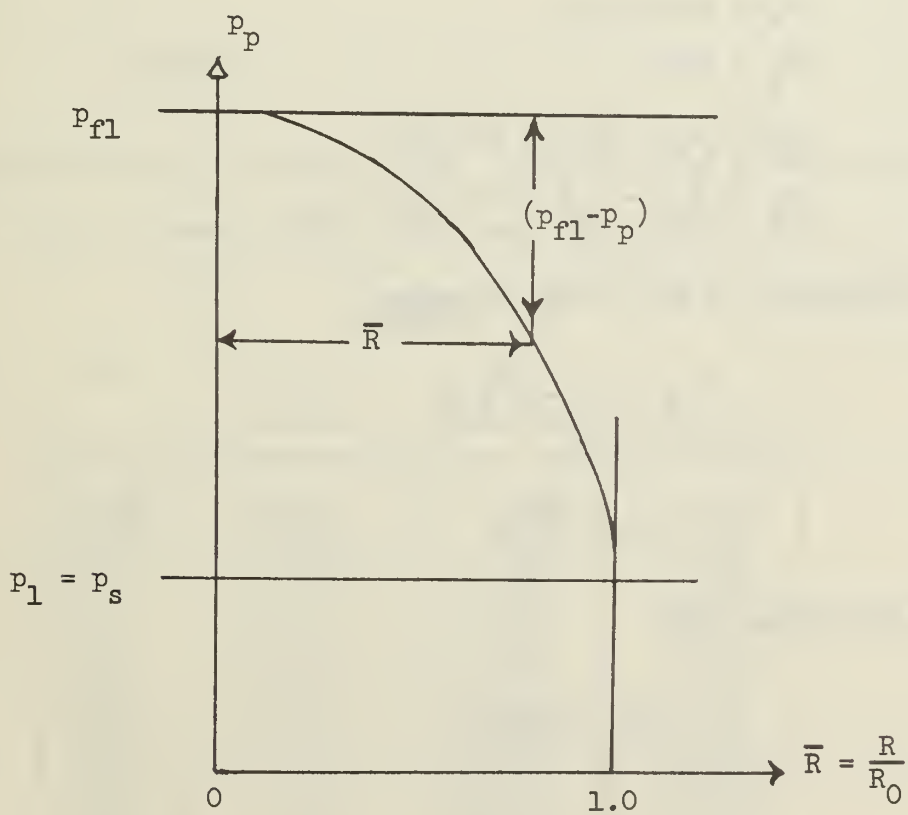
The integral in Eq. 6 is then carried out separately over positive and then negative values of the normalized radius to give two discharge coefficients, C_1 and C_2 . The integration is carried out using an overlapping quadratic technique written as Function Subprogram DATINT, which is described in Appendix B.

The weight flow and Reynolds number are then calculated using the average of C_1 and C_2 , and the average of all values of the upstream flange pressure, p_{fl} and flange to nozzle throat pressure difference, Δ_s . The weight flow is given by Eq. 3 using Eq. 2.

The distributions of F over negative and positive values of the normalized radius are plotted calling Subroutine "DRAW."

Note that to obtain good accuracy, it is necessary to read several values close to the walls. The first input data must be taken at a few thousandths of an inch displacement in from the wall. The last input data similarly must be taken a few thousandths in from the opposite wall. At least one reading should be taken close to the centerline on either side. This is because the integration assumes that the product ($F\ \bar{R}$) goes to zero linearly at the center (where R goes to zero) and also at the wall (where F goes to zero because Δ_1 goes to zero).

V. TEST OF THE DATA REDUCTION PROGRAM (BOX34NØZ)



A parabolic distribution of impact pressure across the nozzle is assumed, as shown in the above sketch. The impact pressure on the centerline is assumed to be equal to the upstream flange pressure, p_{f1} , and the static pressure, p_1 , is taken to be constant and equal to the throat tap pressure, p_s , which is also equal to the downstream flange pressure. Hence,

$$\frac{p_{f1}-p_p}{p_{f1}-p_1} = \bar{\bar{R}}^2 \tag{14}$$

and

$$p_1 = p_s = p_f \tag{15}$$

take the place of probe data. Data assumed to be constant for the test are

$$p_a = 30.00'' \text{ Hg}$$
$$T_t = 70^\circ\text{F}$$
$$\Delta_f \; (= p_a - p_{f1}) = 100'' \; H_2O$$
$$\Delta_s \; (= p_{f1} - p_s) = 20'' \; H_2O = \Delta_{sf}$$

Using the definitions in Eq. 8, Eq. 14 gives

$$\Delta_1 = [1 - \bar{\bar{R}}^2] \; \Delta_s \tag{16}$$

and

$$\Delta_p = \bar{\bar{R}}^2 \; \Delta_s \tag{17}$$

Using Eq. (10) and Eq. (11),

$$p_{f1} = 22.645$$

and

$$p_p = 22.645 - 1.471 \; \bar{\bar{R}}^2$$

Therefore, Eq. (13) gives, for these data without approximation,

$$F = 1.036 \left[1 - 0.03475\ (1 - \bar{R}^2)(1 - 0.06496\ \bar{R}^2)^{-1}\right] \sqrt{(1 - \bar{R}^2)(1 - 0.06496\ \bar{R}^2)}$$

Since $\bar{R}$ is at most unity, the terms $(1 - 0.0646\ \bar{R}^2)^n$ can be expanded to give

$$F \approx 1.036 \left[1-\bar{R}^2\right]^{\frac{1}{2}} \left[1 - 0.03475\ (1-\bar{R}^2)\right] \left[1 - 0.03248\ (1-\bar{R}^2) - 0.03248\right]$$

$$\approx 1.002351 \left[1-\bar{R}^2\right]^{\frac{1}{2}} - 0.001181 \left[1-\bar{R}^2\right]^{3/2} - 0.001171 \left[1-\bar{R}^2\right]^{5/2}$$

From Eq. (6), the coefficient of discharge can be obtained by integration:

$$C = 2 \int_0^1 F\bar{R}d\bar{R} \approx \left[- \frac{1.002351}{1.5} (1-\bar{R}^2)^{3/2} + \frac{0.001181}{2.5} (1-\bar{R}^2)^{5/2} + \frac{0.001171}{3.5} (1-\bar{R}^2)^{7/2}\right]_0^1$$

$$\approx 0.66743$$

The following table lists data values to be input as a test of the computer program. S_0 is taken to be 10 ins., and R_0 = 2.6875 ins.

TABLE 1

$\bar{R}$	Δ_p (EQN 17)	Δ_1 (EQN 16)	R (= $R_0\bar{R}$)
0.995	19.80	.20	2.674
0.990	19.602	.398	2.661
0.98	19.208	.792	2.634
0.95	18.05	1.95	2.553
0.90	16.20	3.80	2.419
0.85	14.45	5.55	2.284
0.80	12.80	7.20	2.150
0.75	11.25	8.75	2.016
0.70	9.80	10.20	1.881
0.65	8.45	11.55	1.747
0.60	7.20	12.80	1.613
0.55	6.05	13.95	1.478
0.50	5.00	15.00	1.344
0.45	4.05	15.95	1.209
0.40	3.20	16.80	1.075
0.35	2.45	17.55	0.941

$\bar{R}$	Δ_p (EQN 17)	Δ_1 (EQN 16)	R ($= R_0\bar{R}$)
0.30	1.80	18.20	0.806
0.25	1.25	18.75	0.672
0.20	0.80	19.20	0.538
0.15	0.45	19.55	0.403
0.10	0.20	19.80	0.269
0.05	0.05	19.95	0.134
0.005	0.00	20.00	0.013

Input Cards:

Card 1: PATMØS = 30.00 ; TATMØS = 70.0 ; SZERØ = 10.0; RZERØ = 2.6875

Card 2: N = 46

Card 3 : S = (10 + COL.4), then (10 - COL.4) of above Table 1

H1 = 0.0

H2 = 100.0

H3 = 0.0

H4 = COL.3 of Table 1 (equal values at $\pm$ R)

H5 = 0.0

H6 = 20.00

H7 = 0.0

H8 = COL.2 of Table 1 (equal values at $\pm$ R)

H9 = 0.0

Card 48 H10 = 20.00

<u>Results</u> from program BOX34NØZ

$C_1 = C_2 = C_{AV} = 0.6671$ [c.f. 0.66743 by analytical integration]

$\dot{m} = 1.9723$ lbs/sec

$R_e = 458555$

VI. APPLICATION

The above method was used to verify the calibration a flow nozzle in the intake duct to the "Hybrid" compressor at the Turbopropulsion Laboratory

(Reference 3). The geometry was that shown schematically in Figure 1. Coefficients of discharge of 0.984 to 0.988 were obtained in four surveys at flow rates ranging from 2.1 to 3.4 lbs/sec. An example of the computer-plotted distribution of the function F for one survey is shown in Figure 2.

REFERENCES

1. Supplement to A.S.M.E. Power Test Codes, Chapter 4, Flow Measurement (Part 5) PTC 19.5; 4-1959, The American Society of Mechanical Engineers, New York.

2. VDI-Durch flutz-Metzregeln. (VDI-Regeln fur die Durch Flutz messung mit genormten Dusen, Blenden und Venturi dusen), 5. Auflage, DIN 1952.

3. Vavra, M. H. and Shreeve, R. P., "A Description of the Turbopropulsion Laboratory in the Aeronautics Department at the Naval Postgraduate School," Naval Postgraduate School Technical Report NPS-57VA72091A, Sept, 1972.

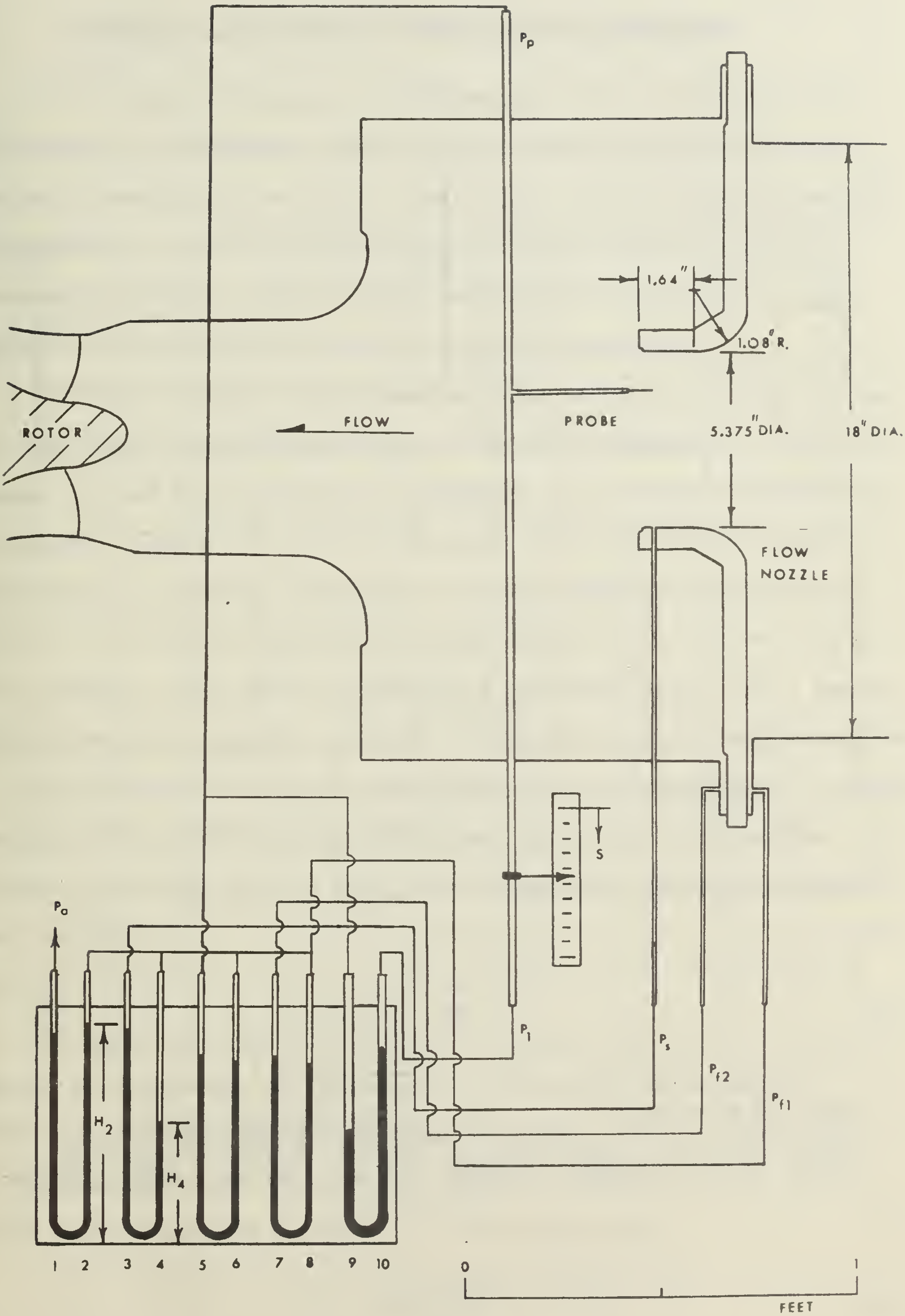


FIGURE 1. SCHEMATIC OF THE INSTRUMENTATION

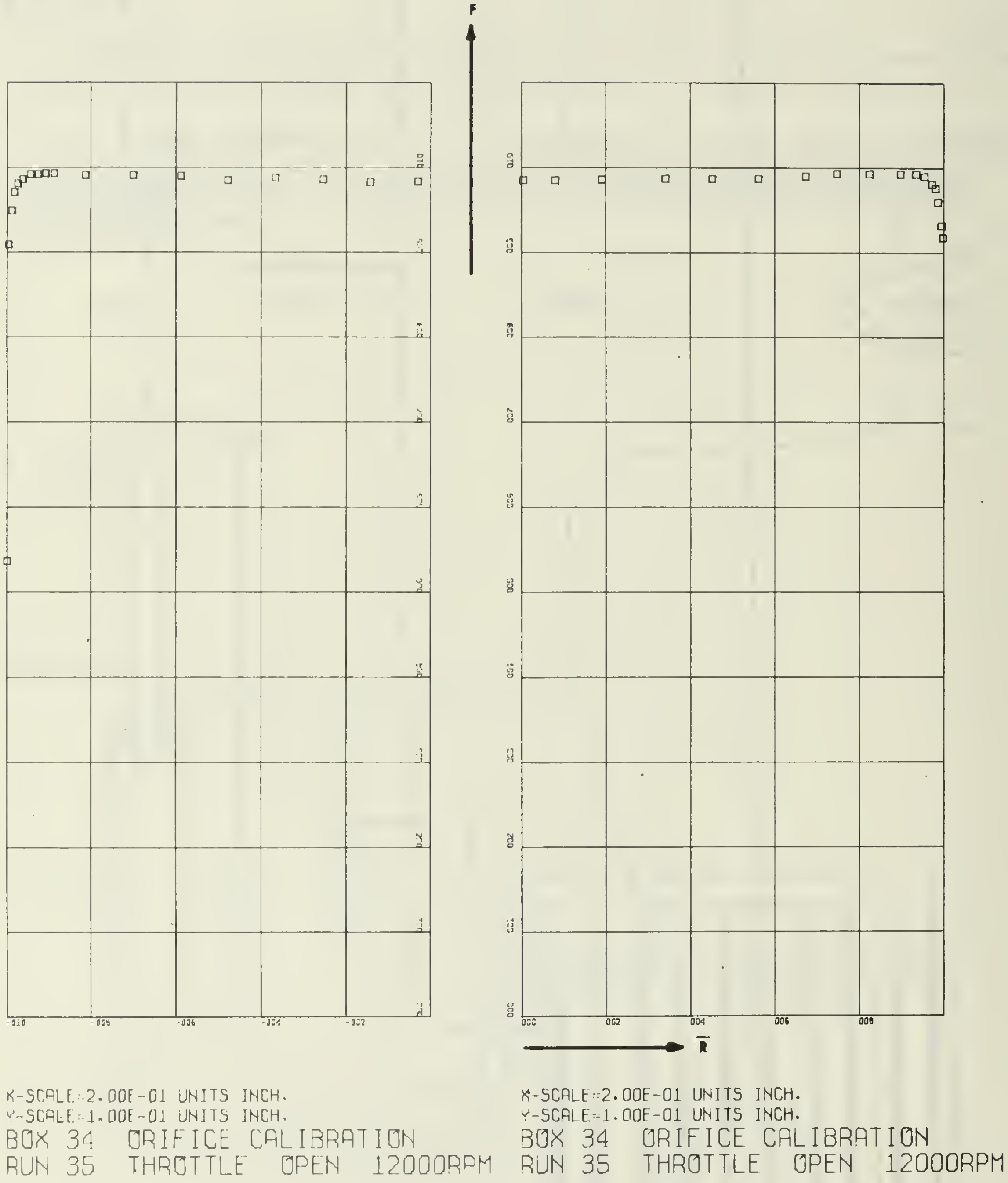


FIGURE 2. EXAMPLES OF COMPUTER PLOTTED DISTRIBUTIONS OF THE RATIO OF THE ACTUAL TO IDEAL MASS FLUX

APPENDIX A. SOME USEFUL COMPRESSIBLE FLOW EXPRESSIONS

In the design and analysis of turbomachines, and in many propulsion problems, it is desirable to deal only with those properties of the flow which of necessity enter into the design, and those which can be measured. Therefore we would like to have equations that relate the velocity to stagnation pressure, static pressure and stagnation pressure, but would choose to avoid the use of density and static temperature.

There are two ways of non-dimensionalizing velocity that avoid the use of the speed of sound, which depends on the static temperature - either the speed of sound at the stagnation temperature (a_t), or the "stagnation" or "limiting" velocity (V_t) can be introduced. While the choice is almost arbitrary, the limiting velocity has a clearly defined significance even for real (non-perfect) gases, and its use results in the tidiest form of the equations. The smaller dimensionless quantities which result, present no problem when a digital calculator or computer is used to perform computations.

The "limiting" velocity is the maximum velocity attainable in an adiabatic expansion from the prevailing state to zero temperature and pressure. The steady-flow energy equation gives, for any gas where gravitational effects are neglected,

$$h_t = h + \frac{V^2}{2} \qquad A(1)$$

where h is enthalpy per unit mass, V is velocity and subscript t, here and hereafter, denotes the stagnation value.

By the above definition, for any gas, $V_t = \sqrt{2h_t}$, but for a gas with constant specific heats, if T is the temperature,

$$V_t = \sqrt{2\, C_p\, T_t} \qquad A(2)$$

For a perfect gas, the stagnation speed of sound, a_t , is given by

$$a_t = \sqrt{\gamma R T_t} = \sqrt{\frac{\gamma-1}{2}} \cdot V_t \qquad A(3)$$

where R is the gas constant. [Note that for air, with $\gamma = 1.402$, $V_t = 109.62 \sqrt{T_t}$ (ft/sec), where T_t is in ^{o}Rankine.]

If the velocity referred to the limiting velocity is defined as

$$X = \frac{V}{V_t} \qquad A(4)$$

reference 1, Page 6, gives the following relationships (which are easily derived) between the local and isentropic-stagnation properties of a perfect gas:

$$\frac{T}{T_t} = 1-X^2 \qquad A(5)$$

$$\frac{p}{p_t} = (1-X^2)^{\frac{\gamma}{\gamma-1}} \qquad A(6)$$

$$\frac{\rho}{\rho_t} = (1-X^2)^{\frac{1}{\gamma-1}} \qquad A(7)$$

where p denotes pressure and ρ denotes density.

By using these expressions, all compressible flow equations can be written in terms of pressures, velocities and stagnation temperature. (Note that in the following development the units are not included. The final expressions must be expressed in consistent units.)

The equation of continuity, giving the mass flow rate ($\dot{m}$) per unit area (A), may be written as

$$\frac{\dot{m}}{A} = \left(\frac{\rho}{\rho_t}\right) \left(\frac{V}{V_t}\right) \rho_t V_t$$

$$= X\,(1-X^2)^{\frac{1}{\gamma-1}} \frac{p_t}{RT_t} \sqrt{2C_p T_t} \qquad A(8)$$

where we have used the perfect gas equation of state, Eq. A(4) and Eq. A(7).

Defining $$F(X) = X\,(1-X^2)^{\frac{1}{\gamma-1}} \qquad A(9)$$

and using the relationship between the gas constant and ratio of specific heats,

$$\frac{\dot{m}}{A} = F(X)\cdot\left(\frac{2\gamma}{\gamma-1}\right)\cdot\frac{p_t}{V_t}\,. \qquad A(10)$$

which gives the mass flux in terms of the velocity, the stagnation pressure and stagnation temperature.

The equation of continuity may also be written as

$$\frac{\dot{m}}{A} = \frac{p}{RT}\left(\frac{V}{V_t}\right)\cdot V_t = \frac{p}{RT_t}\left(\frac{T_t}{T}\right)\left(\frac{V}{V_t}\right)V_t$$

or

$$\frac{\dot{m}}{A} = \left(\frac{X}{1-X^2}\right)\cdot\left(\frac{2\gamma}{\gamma-1}\right)\cdot\frac{p}{V_t} \qquad A(11)$$

which gives the mass flux in terms of the velocity, the static pressure and stagnation temperature.

Equation A(10) and Equation A(11) can be used depending on which quantities are known in any particular case. Equation A(11) can be viewed as a quadratic for the velocity in terms of the static pressure, stagnation temperature and the mass flux. Thus we write A(11) in the form

$$X^2 + 2BX - 1 = 0 \qquad A(12)$$

where the dimensionless quantity, B , is given by

$$B = \left[\left(\frac{\gamma}{\gamma-1}\right)\frac{p}{V_t(\dot{m}/A)}\right] \qquad A(13)$$

If the mass flux, pressure and stagnation temperature are known on a streamline, then the velocity is given by

$$X = \sqrt{B^2+1} - B \qquad A(14)$$

(since the second root is negative).

The measurement of mass flow rate by means of an orifice plate or flow nozzle, depends upon measurements of stagnation pressure and temperature and static pressure. From Eq. A(10), using Eq. A(9) and Eq. A(6), the required form of the continuity equation is then

$$\frac{\dot{m}}{A} = \left(\frac{2\gamma}{\gamma-1}\right) \cdot \frac{p_t}{V_t} \cdot \left(\frac{p}{p_t}\right)^{\frac{1}{\gamma}} \sqrt{\left[1 - \left(\frac{p}{p_t}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad A(15)$$

Using the perfect gas equation of state and the relationship of the specific heat to the gas constant, namely $(\gamma-1)\ C_p = \gamma R$, Eq. A(15) can be written as

$$\frac{\dot{m}}{A} = \rho_t\ V_t \left(\frac{p}{p_t}\right)^{\frac{1}{\gamma}} \sqrt{\left[1 - \left(\frac{p}{p_t}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad A(16)$$

<u>In summary, if the local mass flux is referred to the "stagnation" value, a "total flow function" is obtained which relates the local mass flux to the maximum value of the mass flux attainable from the given state.</u> The "total" flow function, Φ_t , from Eq. A(16) is

$$\Phi_t = \frac{\frac{\dot{m}}{A}}{\rho_t V_t} = \frac{\rho V}{\rho_t V_t} = \left(\frac{p}{p_t}\right)^{\frac{1}{\gamma}} \sqrt{\left[1 - \left(\frac{p}{p_t}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad A(17)$$

Also, using Eq. A(6),

$$\Phi_t = X(1 - X^2)^{\frac{1}{\gamma-1}} = F(X) \qquad A(18)$$

The total flow function given by Eq. A(17) can be approximated for flows at lower speeds by expanding the right hand side of the equation in terms of the difference between the stagnation and static pressures. If $\Delta = p_t - p$, and $\epsilon = \frac{\Delta}{p_t}$ and is small, Eq. A(17) may be written as

$$\Phi_t = \left(1 - \frac{\Delta p}{p_t}\right)^{\frac{1}{\gamma}} \left[1 - \left(1 - \frac{\Delta p}{p_t}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{1}{2}} \qquad A(19)$$

or

$$\Phi_t = \sqrt{\left(\frac{\gamma-1}{\gamma}\right)\epsilon}\left[1 - \frac{3}{4\gamma}\epsilon - \left(\frac{5\gamma+29}{96\gamma^2}\right)\epsilon^2 + 0\left(\epsilon^3\right)\right] \qquad A(20)$$

Using Eq. A(20) in Eq. A(15) and returning to the original notation,

$$\frac{\dot{m}}{A} = \sqrt{\frac{2p_t\left(p_t - p\right)}{RT_t}}\left[1 - \frac{3}{4\gamma}\left(\frac{p_t-p}{p_t}\right) - \left(\frac{5\gamma+29}{96\gamma^2}\right)\left(\frac{p_t-p}{p_t}\right)^2 + 0\left(\ \right)^3\right] \qquad A(21)$$

$$= \begin{matrix}\text{"incompressible"}\\ \text{mass flux}\end{matrix}\ [1 - \text{correction terms}]$$

where again, the units of measurement must be consistent in the use of this expression.

These results are summarized in Table A1.

TABLE A1. COMPRESSIBLE FLOW OF A PERFECT GAS IN TERMS OF THE "LIMITING" (STAGNATION) VELOCITY

DEFINITION	ISENTROPIC RELATIONS	(COMPARISON)
$V_t = \sqrt{2C_pT_t}$ $(= \sqrt{2h_t}$ for real gas$)$ $X = V/V_t$	$T/T_t = 1-X^2$ $p/p_t = \left(1-X^2\right)^{\frac{\gamma}{\gamma-1}}$ $\rho/\rho_t = \left(1-X^2\right)^{\frac{1}{\gamma-1}}$	$M = \frac{V}{\sqrt{\gamma RT}} = \sqrt{\frac{2}{\gamma-1}} \cdot \frac{X}{\sqrt{1-X^2}}$ $N = \frac{V}{\sqrt{\gamma RT_t}} = \sqrt{\frac{2}{\gamma-1}} \cdot X$

FORMS OF THE CONTINUITY EQUATION, KNOWING:

	V,p_t,T_t [Eq. A(10)]	V,p,T_t [Eq. A(11)]	p,p_t,T_t [Eq. A(15)]
$\frac{\dot{m}}{A} =$	$\left(\frac{2\gamma}{\gamma-1}\right) \cdot \frac{p_t}{V_t} \cdot X\left(1-X^2\right)^{\frac{1}{\gamma-1}}$	$\left(\frac{2\gamma}{\gamma-1}\right) \cdot \frac{p}{V_t} \cdot \left(\frac{X}{1-X^2}\right)$	$\left(\frac{2\gamma}{\gamma-1}\right) \cdot \frac{p}{V_t} \cdot \left(\frac{p}{p_t}\right)^{\frac{1}{\gamma}} \sqrt{\left[1-\left(\frac{p}{p_t}\right)^{\frac{\gamma-1}{\gamma}}\right]}$

FORMS OF THE "TOTAL" FLOW FUNCTION

$$\Phi_t = \frac{\rho V}{\rho_t V_t} \; , \; \frac{\dot{m}/A}{\rho_t V_t} \; , \; F(X) = X\left(1 - X^2\right)^{\frac{1}{\gamma-1}} \; , \; \left(\frac{p}{p_t}\right)^{\frac{1}{\gamma}} \sqrt{\left[1 - \left(\frac{p}{p_t}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

SOLUTION OF THE CONTINUITY EQUATION FOR X:

$$X = \sqrt{B^2 + 1} - B \; , \text{ from } X^2 + 2BX - 1 = 0$$

$$\text{where } B = \left[\left(\frac{\gamma}{\gamma-1}\right) \frac{p}{V_t(\dot{m}/A)}\right]$$

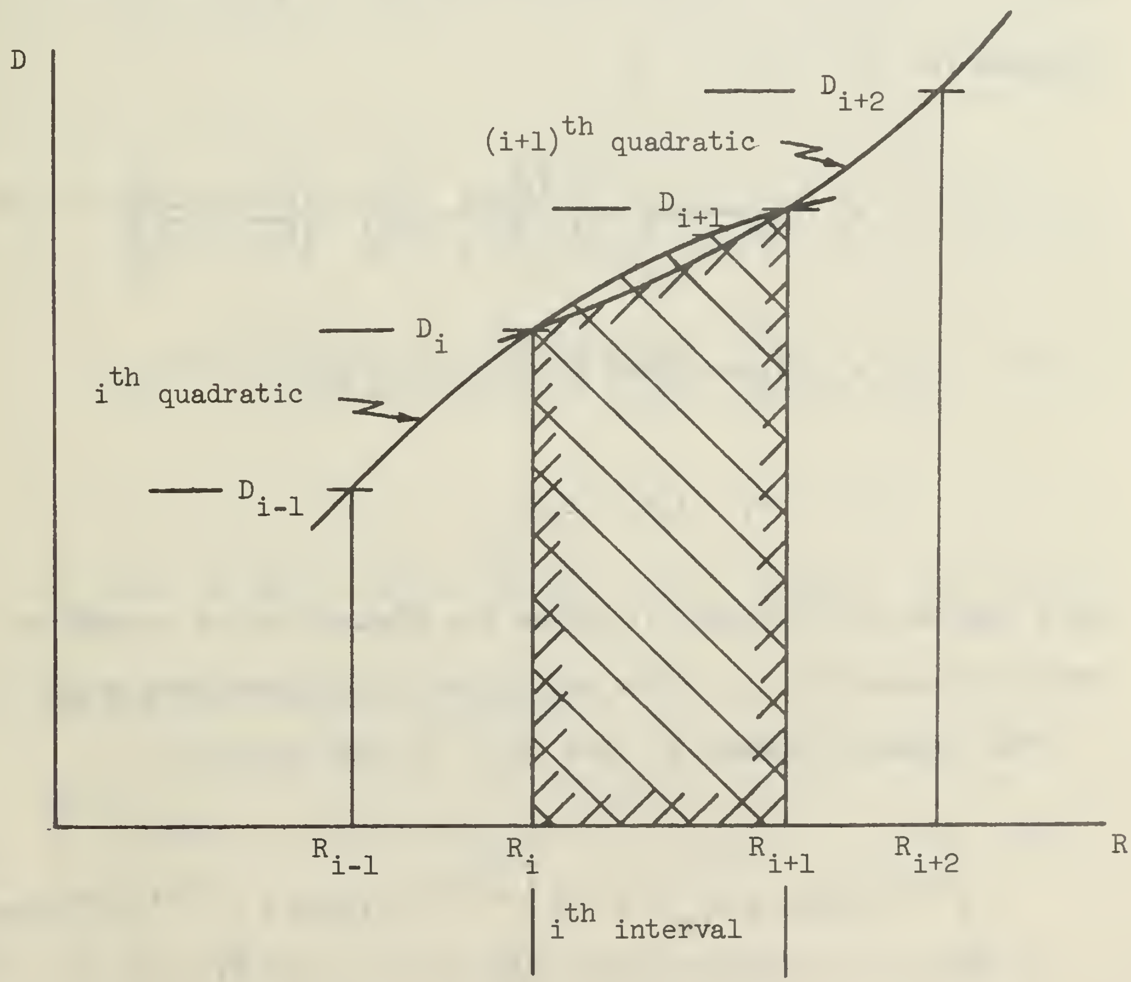
APPROXIMATION OF THE MASS FLUX FOR LOW VELOCITIES:

$$\frac{\dot{m}}{A} = \sqrt{\frac{2p_t(p_t-p)}{RT_t}} \left[1 - \frac{3}{4\gamma}\left(\frac{p_t-p}{p_t}\right) - \left(\frac{5\gamma+29}{96\gamma^2}\right)\left(\frac{p_t-p}{p_t}\right)^2 + O(\;)^3\right]$$

APPENDIX B. COMPUTER INTEGRATION OF DATA GIVEN AT ARBITRARY INTERVALS

Data, D_i , are given at arbitrary values of a variable, R_i . The data approximate an unknown function $\bar{D}(R)$. An approximation of the integral , $\int \bar{D}(R)dR$, is required over the interval R_1 to R_{N+1} .

The method is to define quadratic curves through each three adjacent data points and to approximate the integral between data points by the average of the areas under the two overlapping quadratics defined in that interval. This is illustrated for the i^{th} interval between R_i and R_{i+1} in the following figure.



The i^{th} quadratic passing through (R_{i-1}, D_{i-1}) , (R_i, D_i) and (R_{i+1}, D_{i+1}) can be written generally as

$$D^i(R) = A_i R^2 + B_i R + C_i \tag{1}$$

where A_i , B_i and C_i are to be determined from the three equations obtained by inserting the known points:

$$\left.\begin{aligned} D_{i-1} &= A_i R_{i-1}^2 + B_i R_{i-1} + C_i \\ D_i &= A_i R_i^2 + B_i R_i + C_i \\ D_{i+1} &= A_i R_{i+1}^2 + B_i R_{i+1} + C_i \end{aligned}\right\} \tag{2}$$

Solving for A_i , B_i , C_i ,

$$A_i = \left(\frac{1}{R_{i+1} - R_{i-1}}\right) \left[\left(\frac{D_{i+1} - D_i}{R_{i+1} - R_i}\right) - \left(\frac{D_i - D_{i-1}}{R_i - R_{i-1}}\right)\right] \tag{3}$$

$$B_i = \left(\frac{D_i - D_{i-1}}{R_i - R_{i-1}}\right) - A_i \,(R_i + R_{i-1}) \tag{4}$$

$$C_i = D_i - A_i R_i^2 - B_i R_i \tag{5}$$

Note that Eq (3), (4) and (5) define the complete set of quadratics, where the subscript i refers the center of the three data points.

The integral between R_i and R_{i+1} is then given by

$$\int_{R_i}^{R_{i+1}} \bar{D}(R)dR \approx D_{int}(i) = \frac{1}{2}\left[\int_{R_i}^{R_{i+1}} D^i(R)dR + \int_{R_i}^{R_{i+1}} D^{i+1}(R)dR\right] \tag{6}$$

Using Eq (1) and integrating analytically,

$$\int_{R_i}^{R_{i+1}} D^i(R)dR = \frac{A_i}{3}\left(R_{i+1}^3 - R_i^3\right) + \frac{B_i}{2}\left(R_{i+1}^2 - R_i^2\right) + C_i\left(R_{i+1} - R_i\right)$$

and

$$\int_{R_i}^{R_{i+1}} D^{i+1}(R)dR = \frac{A_{i+1}}{3}\left(R_{i+1}^3 - R_i^3\right) + \frac{B_{i+1}}{2}\left(R_{i+1}^2 - R_i^2\right) + C_{i+1}\left(R_{i+1} - R_i\right)$$

so that

$$D_{int}(i) = \left(\frac{A_{i+1} + A_i}{6}\right)\left(R_{i+1}^3 - R_i^3\right) + \left(\frac{B_{i+1} + B_i}{4}\right)\left(R_{i+1}^2 - R_i^2\right) + \left(\frac{C_{i+1} + C_i}{2}\right)\left(R_{i+1} - R_i\right) \tag{7}$$

Over the first and last intervals, only one quadratic can be defined, so that

$$D_{int}(1) = \frac{A_2}{3}\left(R_2^3 - R_1^3\right) + \frac{B_2}{2}\left(R_2^2 - R_1^2\right) + C_2\left(R_2 - R_1\right) \tag{8}$$

and

$$D_{int}(N) = \frac{A_N}{3}\left(R_{N+1}^3 - R_N^3\right) + \frac{B_N}{2}\left(R_{N+1}^2 - R_N\right) + C_N\left(R_{N+1} - R_N\right) \tag{9}$$

Finally, the complete integral is given by

$$\int_{R_i}^{R_{N+1}} \bar{D}(R)dR \approx \text{"DATINT"} = D_{int}(1) + \sum_{i=2}^{N-1} D_{int}(i) + D_{int}(N) \tag{10}$$

Eqs (3), (4), (5), (7), (8), (9) and (10) were programmed in Fortran IV as Function Subprogram DATINT. The program takes N+1 consecutive values of R and D from the main program and returns the value of the integral.

The notation is similar to the above description.

PROGRAM LISTING:

```
      FUNCTION DATINT(R,D,NP1)
      DIMENSICN R(NP1),D(NP1)
      DIMENSION AI(50),BI(50),CI(50),DINT(50)
      N=NP1-1
      NM1=N-1
      DC 200 I=2,N
      AI(I)=(1.0/(R(I+1)-R(I-1)))*((D(I+1)-D(I))/(R(I+1)-R(I))-(D(I)-
     1      D(I-1))/(R(I)-R(I-1)))
      BI(I)=(D(I)-D(I-1))/(R(I)-R(I-1))-(R(I)+R(I-1))*AI(I)
      CI(I)=D(I)-AI(I)*R(I)**2-BI(I)*R(I)
  2CC CONTINUE
      DATINT=0.0
      DC 210 I=2,NM1
      DINT(I)=(AI(I)+AI(I+1))*(R(I+1)**3-R(I)**3)/6.0+(BI(I)+BI(I+1))*
     1   (R(I+1)**2-R(I)**2)/4.0+(CI(I)+CI(I+1))*(R(I+1)-R(I))/2.0
      DATINT=DATINT+DINT(I)
  210 CCNTINUE
      DINT(1)=AI(2)*(R(2)**3-R(1)**3)/3.0+BI(2)*(R(2)**2-R(1)**2)/2.0
     1       +CI(2)*(R(2)-R(1))
      DINT(N)=AI(N)*(R(N+1)**3-R(N)**3)/3.0+BI(N)*(R(N+1)**2-R(N)**2)/
     1       2.0+CI(N)*(R(N+1)-R(N))
      DATINT=DATINT+DINT(1)+DINT(N)
      RETURN
      END
```

APPENDIX C. NOZZLE CALIBRATION DATA REDUCTION PROGRAM

```
// EXEC FORTCLGP,REGION.GO=100K
//FORT.SYSIN DD *
C THIS IS PROGRAM NOZ. IT ACCEPTS RAW CATA FROM SURVEYS OF A FLOW
C MEASURING NOZZLE, AND COMPUTES THE COEFFICIENT CF DISCHARGE,THE WEIGHT
C FLCW AND THE REYNOLDS NUMBER.
C
      DIMENSION S(50),H1(50),H2(50),H3(50),H4(50),H5(50),H6(50),H7(50),
     1H8(50),PP(50),PF(50),D1(50),DS(50),DP(50),DF(50),F(50),F2(50),
     2R(50),R2(50),FR1(50),FR2(50),H9(50),H10(50)
      REAL*8 LABEL/'        '/
      REAL*8 TITLE(12)/'BOX 34  ',' NOZZLE','CALIBRAT','ION     ',2*'
     1        ','RUN 35  ','THROTTLE','  OPEN  ','12COORPM',2*'        '/
C
C READ CATA
C
      READ (5,1) NDAY,NMTH,NYR,NSCAN,NRPM, ORFICE,PATMOS,TATMOS,SZERO ,
     1                                                          RZERO
      READ (5,2) N
    1 FORMAT (I2,I2,I2,I4,2I10,F10.2,F10.1,F10.3,F10.4)
    2 FORMAT (I2)
      READ (5,3) (S(I),H1(I),H2(I),H3(I),H4(I),H5(I),H6(I),H7(I),H8(I),
     1H9(I),H10(I),I=1,N)
    3 FCRMAT (F10.3,10F6.2)
C
C CALCULATE THE INTEGRANC AT EACH POINT
C
      DO 5 I=1,N
      DS(I)=H3(I)-H4(I)
      DP(I)=H5(I)-H6(I)
      D1(I)=H10(I)-H9(I)
      DF(I)=H2(I)-H1(I)
      PF(I)=PATMOS-0.07355*DF(I)
      PP(I)=PF(I)-0.07355*DP(I)
      F(I)=FINTG(D1(I),DS(I),PP(I),PF(I))
      WRITE (6,4)F(I)
    4 FORMAT ('0',F10.6)
    5 CONTINUE
C
C NOW CALCULATE THE RADIUS AT EACH POINT THEN C1 AND C2.
C
      PI=3.14159265
      AZERO=PI*RZERO*RZERO
      DC 25 I=1,N
      R(I)=(S(I)-SZERO)/RZERO
      IF (R(I))20,10,10
   1C FR1(I)=F(I)*R(I)
      INDEX=I
      GC TO 25
```

```
   20 J=I-INDEX
      R2(J)=R(I)
      F2(J)=F(I)
      FR2(J)=F2(J)*R2(J)
      WRITE (6,4) FR2(J)
   25 CONTINUE
      N2=N-INDEX
   26 WRITE (6,27)
   27 FORMAT ('0','     RADIUS      F        F*RADIUS')
   28 WRITE (6,29) ((R(I), F(I),FR1(I)),I=1,INDEX)
   29 FORMAT ('0',3F10.4)
   30 WRITE (6,29) ((R2(I),F2(I),FR2(I)),I=1,N2)
      AINT=DATINT (R,FR1,INDEX)
      C1=(1.0-R(1))*FR1(1)-2.0*AINT+R(INDEX)*FR1(INDEX)
      BINT=DATINT (R2,FR2,N2)
      C2=R2(1)*FR2(1)+2.0*BINT-(R2(N2)+1.0)*FR2(N2)
C
C PROGRAM NOW CALCULATES WEIGHT FLOW AND REYNOLCS NUMBER USING C AVERAGE
      C12=(C1+C2)/2.0
      SUM1=0.0
      SUM2=0.0
      DO 35 I=1,N
      SUM1=SUM1+PF(I)
      SUM2=SUM2+DS(I)
   35 CONTINUE
      PTAV=SUM1/N
      DTAV=SUM2/N
      WTC=0.146033*C12*AZERO *SQRT(PTAV*DTAV/(TATMCS+460.0))*
     1  (1.0-0.03935*DTAV/PTAV)
      VIS=VRATIO(TATMOS)*1.153E-5
      RE=WTC*(RZERO*24.0/AZERO)/VIS
C
C NOW PRINT OUT THE RESULTS
      WRITE (6,40)
   40 FORMAT ('1','HYBRID COMPRESSOR FLOW ORIFICE CALIBRATION TEST RESUL
     1TS        DATE')
      WRITE (6,41) NSCAN,NDAY,NMTH,NYR
   41 FCRMAT ('0',I47,I13,I3,I3)
      WRITE (6,42)
   42 FORMAT ('0','  RPM     ORIFICE      C1         C2        C(AVG)     WEIGH
     1T FLOW       REY NO')
      WRITE (6,43) NRPM,ORFICE,C1,C2,C12,WTC,RE
   43 FCRMAT ('0',I5,I11,2F9.4,F10.4,F15.4,F12.C)
      CALL DRAW(17,R,F,0,3,LABEL,TITLE,0.2,0.1,0,0,2,2,5,11,1,LAST)
      CALL DRAW(18,R2,F2,0,3,LABEL,TITLE,0.2,0.1,0,5,2,2,5,11,1,LAST)
      STCP
      END
C FUNCTION SUBPROGRAM TO INTEGRATE THE CATA
```

```
      FUNCTION DATINT(R,D,NP1)
      DIMENSION R(NP1),D(NP1)
      DIMENSION AI(50),BI(50),CI(50),DINT(50)
      N=NP1-1
      NM1=N-1
      DC 200 I=2,N
      AI(I)=(1.0/(R(I+1)-R(I-1)))*((D(I+1)-D(I))/(R(I+1)-R(I))-(D(I)-
     1      D(I-1))/(R(I)-R(I-1)))
      BI(I)=(D(I)-D(I-1))/(R(I)-R(I-1))-(R(I)+R(I-1))*AI(I)
      CI(I)=D(I)-AI(I)*R(I)**2-BI(I)*R(I)
  200 CONTINUE
      CATINT=0.0
      DO 210 I=2,NM1
      DINT(I)=(AI(I)+AI(I+1))*(R(I+1)**3-R(I)**3)/6.0+(BI(I)+BI(I+1))*
     1    (R(I+1)**2-R(I)**2)/4.0+(CI(I)+CI(I+1))*(R(I+1)-R(I))/2.0
      DATINT=DATINT+DINT(I)
  210 CCNTINUE
      DINT(1)=AI(2)*(R(2)**3-R(1)**3)/3.0+BI(2)*(R(2)**2-R(1)**2)/2.0
     1       +CI(2)*(R(2)-R(1))
      DINT(N)=AI(N)*(R(N+1)**3-R(N)**3)/3.0+BI(N)*(R(N+1)**2-R(N)**2)/
     1      2.0+CI(N)*(R(N+1)-R(N))
      DATINT=DATINT+DINT(1)+DINT(N)
      WRITE (6,190) (AI(I),BI(I),CI(I),DINT(I),I=2,N)
  19C FORMAT (4F10.4)
      WRITE (6,191) DATINT
  191 FORMAT (F10.4)
      RETURN
      END
C FUNCTICN SUBPROGRAM TO EVALUATE THE INTEGRAND AT EACH CATA POINT
C


      FUNCTION FINTG(DEL1,DELS,PP,PF)
      GAMMA=1.402
      A=(1.0-(0.75*DEL1*0.07355)/(GAMMA*PP))
      B=(1.0-(0.75*DELS*0.07355)/(GAMMA*PF))
      FINTG=SQRT(PP*DEL1/(PF*DELS))*A/B
      RETURN
      END
C FUNCTICN SUBPROGRAM TC CALCULATE VISCOSITY RATIC FROM SUTHERLANDS LAW
C

      FUNCTION VRATIO (TEMP)
      VRATIO=0.06333*SQRT(TEMP+460.0)/(198.72/(TEMP+460.0)+1.0)
      RETURN
      END
//GO.SYSIN DD *
```

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13. ABSTRACT

A description is given of a method for calibrating flow nozzles used in compressor testing which is insensitive to flow variations during a survey. A computer program to reduce hand-recorded data from manometers is described which uses a compressible flow expression whose derivation is included in the first appendix. This appendix gives forms of the statement of continuity along a streamline when "limiting" values of the density and velocity are used to non-dimensionalize these parameters. This technique has been found to simplify the writing of computer programs for analyzing measurements from turbomachines, and to generally simplify analyses of flows that can be taken to be adiabatic.

The method used to integrate experimental data taken over arbitrary intervals, which is of general use, is described and a listing of the subroutine is given in a second appendix.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Flow measurement						
Flow rate						
Flow calibration						
Compressible flow						
Compressors						
Turbomachinery						